# Sheep Collisions: the Good, the Bad, and the TBI

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February 2, 2008

#### Abstract

The title page of Chapter 9 in **Fundamentals of Physics** (Halliday, Resnick, and Walker,  $8^{th}$  Edition, p. 201) shows a dramatic photograph of two Big Horn sheep butting heads and promises to explain how sheep survive such violent clashes without serious injury. However, the answer presented in sample problem 9-4 (p. 213) errs in presuming an interaction time of 0.27s which results in an unrealistically long stopping distance of 0.62m. Furthermore, the assertion that the horns provide necessary cushioning of the blow is inconsistent with the absence of concussions in domestic breeds of hornless sheep. Results from traumatic brain injury (TBI) research allow acceleration tolerance of sheep to be estimated as 450g facilitating an analysis of sheep collisions that is more consistent with available observations (stopping distance less than 1cm, impact time of roughly 2ms).

## I. ERRANT EXAMPLE IN POPULAR BOOK

Sample problem 9-4 in a popular textbook stands out because the title page of Chapter 9 shows a dramatic photograph of two Big Horn sheep butting heads and asserts that the chapter will describe how the sheep can survive such violent clashes without falling to the ground with concussions.[1] Jearl Walker also uses this photo and example in speaking engagements entitled *The Flying Circus of Physics*.[2] However, the answer presented in Sample Problem 9-4 is incorrect. The physics seems sound, but the flaw in presuming

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an interaction time of  $t_f = .27s$  becomes clear when the resulting stopping distance is computed.

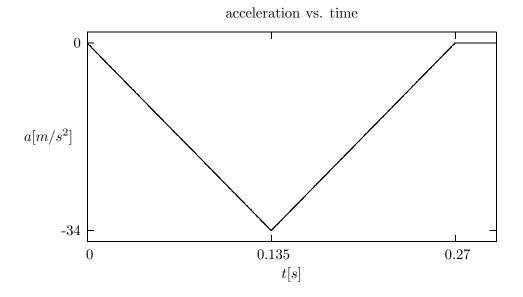


Figure 1: Acceleration of Big Horn sheep given in sample problem.

Fig. 1 shows the acceleration given in the sample problem. Time t=0s corresponds to the beginning of the collision. Letting  $a_{max}=34m/s^2$ , the acceleration decreases linearly from t=0s until  $t_1=\frac{t_f}{2}=0.135s$  as

$$a_1(t) = -\frac{2a_{max}}{t_f}t. (1)$$

Then the acceleration increases linearly from  $t_1=0.135s$  to  $t_f=0.27s$  as

$$a_2(t) = -2a_{max} + \frac{2a_{max}}{t_f}t.$$
 (2)

The sheep each have a mass of 90kg so that F(t) is the mass times the acceleration function. The impulse is calculated to be the area under the force curve or  $-\frac{1}{2}ma_{max}t_f=-413N\cdot s$ . Since this is the impulse that brings one sheep to a stop, it is equal and opposite to the initial momentum of the sheep,  $p_i=\frac{1}{2}ma_{max}t_f=413kg\cdot m/s$ , which implies an initial velocity,  $v_i=p_i/m=\frac{1}{2}a_{max}t_f=4.589m/s$ . (One can also determine  $v_i$  from the average acceleration  $a_{ave}=-17m/s^2$  and the time interval 0.27s.)

The explanation in the "Comment:" portion of the sample problem asserts that the horns of the sheep cushion the impact by increasing the duration of the collision to  $t_f=0.27s$ . Supposedly, injury is prevented, because the long duration reduces the forces to a level that the sheep can tolerate. However, for an impact duration of  $t_f=0.27s$ , the horns would have to extend forward beyond the skull for a distance at least as large as the stopping distance, which we will show is unrealistic.

### A. Velocity from integrating the acceleration

The velocity decreases from t = 0s until  $t_1 = 0.135s$  as

$$v_1(t) = v_i + \int_0^t a_1(t)dt = v_i + \int_0^t \frac{-2a_{max}}{t_f} tdt$$
$$= \frac{1}{2} a_{max} t_f - \frac{a_{max}}{t_f} t^2.$$
(3)

Evaluating at  $t=t_1=\frac{t_f}{2}=0.135s$  gives  $v_{2i}=\frac{1}{4}a_{max}t_f=2.295m/s$ . The velocity then decreases from  $t_1=0.135s$  to  $t_f=0.27s$  as

$$v_{2}(t) = v_{2i} + \int_{t_{1}}^{t} a_{2}(t)dt = v_{2i} + \int_{t_{1}}^{t} \left(-2a_{max} + \frac{2a_{max}}{t_{f}}t\right)dt$$

$$v_{2}(t) = v_{2i} + 2a_{max}\frac{t_{f}}{2} - \frac{a_{max}}{t_{f}}\left(\frac{t_{f}}{2}\right)^{2} - 2a_{max}t + \frac{a_{max}}{t_{f}}t^{2}.$$
(4)

which substituting in  $v_{2i} = \frac{1}{4}a_{max}t_f = 2.295m/s$  simplifies to

$$v_2(t) = a_{max}t_f - 2a_{max}t + \frac{a_{max}}{t_f}t^2.$$
 (5)

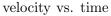
As expected,  $v_2(t) = 2.295 m/s$  at  $t = \frac{t_f}{2} = 0.135 s$ , and  $v_2(t) = 0 m/s$  at  $t = t_f = 0.27 s$ .

Fig. 2 shows the velocity of one sheep. As expected, the velocity decreases monotonically from t=0s until  $t_f=0.27s$ . However, unlike many introductory physics problems (those with constant acceleration), the velocity is not a linear function of time.

#### B. Position from integrating the velocity

The displacement during impact increases from t = 0s to  $t_1 = \frac{t_f}{2} = 0.135s$  as

$$x_1(t) = v_i t + \int_0^t \frac{-a_{max}}{t_f} t^2 dt = \frac{1}{2} a_{max} t_f t - \frac{a_{max}}{3t_f} t^3.$$
 (6)



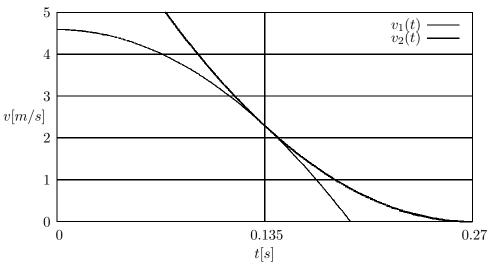


Figure 2: Velocity of sheep during collision given model acceleration. Note that the velocity is  $v_1(t)$  for  $0 \le t \le 0.135s$  and  $v_2(t)$  for  $0.135s \le t \le 0.27s$ .

Evaluating this at  $t = t_1 = \frac{t_f}{2} = 0.135s$  gives  $x_{2i} = \frac{5}{24}a_{max}t_f^2 = 0.5162m$ .

The position continues to increase from  $t_1 = \frac{t_f}{2} = 0.135s$  to  $t_f = 0.27s$  as

$$x_{2}(t) = x_{2i} + \int_{t_{1}}^{t} \left( a_{max} t_{f} - 2a_{max} t + \frac{a_{max}}{t_{f}} t^{2} \right) dt$$

$$x_{2}(t) = -\frac{1}{12} a_{max} t_{f}^{2} + a_{max} t_{f} t - a_{max} t^{2} + \frac{a_{max}}{3t_{f}} t^{3}.$$
(7)

 $x_1(t)$  and  $x_2(t)$  are shown in Fig. 3. Evaluating  $x_2(t)$  at  $t=t_f$  gives the total stopping distance:

$$x_2(t_f) = \frac{1}{4}a_{max}t_f^2 = \frac{1}{2}v_i t_f = 0.6195m.$$
(8)

Fig. 3 shows the displacement after impact.

#### C. Discussion of concussion and brain injury

The assertion that horns cushion the blow by spreading out the force over a time interval of 0.27s results in an unrealistic stopping distance of 0.6195m,

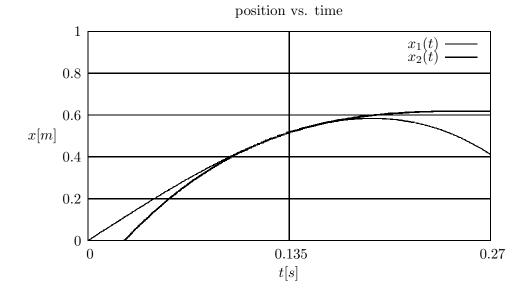


Figure 3: Position of sheep during collision given model acceleration. Note that the position is  $x_1(t)$  for  $0 \le t \le 0.135s$  and  $x_2(t)$  for  $0.135s \le t \le 0.27s$ .

because the horns do not extend forward or flex sufficiently to provide this cushion. The stopping distance provided by the horns is likely much smaller resulting in much larger forces and accelerations.

It is well recognized in the traumatic brain injury (TBI) literature[3] that certain animals(such as woodpeckers and head-butting ruminants) are much more resistant than humans to TBI and that this resistance to TBI is not simply explained by introductory mechanics.[4] The detailed biomechanical reasons for this resistance to TBI are not well-understood, but there is some evidence to suggest that ruminants that engage in pre-mating head butting rituals are not only more resistant to TBI from externally imposed forces, accelerations, and blunt-force trauma. They are also more resistant to TBI induced from an internal ballistic pressure wave originating in the thoracic cavity.[5] This suggests that resistance to TBI in these species has a neurological as well as a biomechanical basis.

#### D. A simpler approach

Few textbook readers are likely to perform the above integrations necessary to determine the stopping distance. However, a reasonable estimate of the stopping distance is possible simply using the average acceleration and assuming (wrongly, of course) that the accleration is constant and estimating the stopping distance from kinematics (or the Work-Energy theorem).

Since  $v_i = 4.589m/s$  and  $v_f = 0m/s$ , assuming a constant acceleration gives  $v_{ave} = v_i/2 = 2.295m/s$ . The estimated stopping distance is  $x = v_{ave}t_f = 0.6195m$ . Why is the stopping distance estimated from an erroneous assumption the same as the rigorously computed stopping distance? The symmetry of the acceleration about t = 0.135s results in a symmetry in v(t) so that  $v_{ave} = v_i/2 = 2.295m/s$  even though the acceleration is not constant. This is a happy accident that could not be inferred with confidence without well-developed intuition.[6] However, estimates of stopping distances from assuming a constant acceleration are often within a factor of two of rigorous calculations.

#### II. A MORE REASONABLE ANALYSIS

Having raised domestic sheep, we have observed that a number of horn-less sheep breeds engage in pre-mating dominance rituals of head-butting without falling to the ground with concussions.[7] This is also observed by many shepherds. If breaking off a horn resulted in great danger to Big Horn sheep because horns are critical in spreading out the momentum change so that the forces and accelerations are smaller,[1, Sample Problem 9-4] then a higher incidence of concussion would be seen in the hornless sheep breeds.

Head butting ruminants are more resistant to concussion than humans, but firm quantitative thresholds for tramautic brain injury (TBI) in humans remain a matter of debate and ongoing research. Both motorcycle and combat helmet standards can be expressed in terms of allowable model head accelerations for a specified impact velocity under certain test conditions. (A moving model head wearing the helmet impacts a stationary object and the head acceleration is measured.)

For example, the US Army's standard for their advanced combat helmet is 150g when impacting a hard surface at 10 ft/s.[8] The Department of Transportation (DOT) standard for motorcycle helmets is roughly 250g for high-energy impacts and most DOT approved helmets produce under 200g for commonly encountered impacts.[9] An oft-cited car crash study estimated the acceleration tolerance in humans as 220g for impacts with a duration of 2ms[10]. While it is not well established that these acceleration thresholds accurately predict the presence or absence of mild TBI, these standards represent an accepted range of acceleration tolerance in humans.[11]

Using the approximation of constant acceleration, combining an acceleration threshold with an impact velocity allows a computation of a minimum stopping distance from kinematics:

$$d = \frac{v_i^2}{2a},\tag{9}$$

where d is the stopping distance,  $v_i$  is the impact velocity, and a is the acceleration threshold. For an acceleration of 150g and an impact velocity of 10ft/s (2.5m/s), the minimum stopping distance is 0.01ft (0.25cm).

The impact velocity of Big Horn sheep might be as large as  $v_i = 8.9m/s$ .[12] The acceleration tolerance in sheep can be estimated as 450g, which is consistent with the Gibson scaling rule[4]. This approach yields a minimum stopping distance of roughly 0.009m. This is much smaller than the sample problem's stopping distance of 0.62m, and can be accommodated even in hornless sheep. This suggests the greater resistance of sheep to TBI (higher acceleration threshold) rather than horns is the dominant factor in sheep survival of head butting collisions.[13] This analysis implies an interaction time of roughly 0.002s, much shorter than the sample problem interaction time of 0.27s.

#### III. CONCLUSION

In conclusion, both the theoretical analysis (computation of stopping distance) and the experimental evidence (observations in hornless domestic sheep) contradict the assertion that the sheep's resistance to concussion is related to the horn spreading out the impulse over a time interval as long as 0.27s. Stopping distances shorter than 0.01m are consistent with reasonable estimates of acceleration threshold and impact velocity. There may be biomechanical properties within the sheep heads that provide for a cushion of roughly 1cm, but their resistence to concussions also seems to have a neurological basis. The question of sheep resistance to head butting injury makes for an interesting and illuminating example in introductory Physics, but realistic values of the interaction time and distance must be used to compute reasonable values of the forces and accelerations involved.

#### References

[1] David Halliday, Robert Resnick, Jearl Walker, Fundamentals of Physics (Wiley, New York, 2007), 8<sup>th</sup> ed. p. 201, p. 213.

- [2] There is also a qualitatitative description in the book by the same name. Jearl Walker, *The Flying Circus of Physics*, (Wiley, New York, 2006), 2<sup>nd</sup> ed.
- [3] Nigel A. Shaw, "The Neurophysiology of Concussion," Progress in Neurobiology **67**, 281-344 (2002).
- [4] L.J. Gibson, "Woodpecker pecking: How woodpeckers avoid brain injury," Journal of Zoology **270**, 462-465 (2006). Small brain size, short impact duration, and brain orientation within the skull combine to contribute to woodpecker acceleration tolerance.
- [5] See Figure 6, Michael Courtney, Amy Courtney, "Ballistic pressure wave contributions to rapid incapacitation in the Strasbourg goat tests," arxiv.org/ftp/physics/papers/0701/0701267.pdf
- [6] It is a general result that  $v_{ave} = v_i/2$  for stopping problems where the acceleration is symmetric about the midpoint in time.
- [7] Krouse P, "Young Growers Go After Niche of Selling Right to Consumers," Cleveland Plain Dealer, October 23, 2002, p. C1. The sheep pictured are two hornless rams that regularly engaged in head-butting. Also see: Better Physics Through Farming at www.ballisticstestinggroup.org/mwc.htm
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- [11] Studies of helmet to helmet collisions in professional football suggest a threshold range of 66g 106g for impacts of roughly 10ms duration.

- Liying Zhang, King H. Yang, and Albert I. King, "A Proposed Injury Threshold for Mild Traumatic Brain Injury," *Journal of Biomechanical Engineering* **126**, 226-236 (2004).
- $[12] \begin{tabular}{ll} See & animals.nationalgeographic.com/animals/mammals/rocky-mountain-bighorn-sheep.html \\ \end{tabular}$
- [13] These estimates are rough, but at least represent collision parameters consistent with each other and available observations.